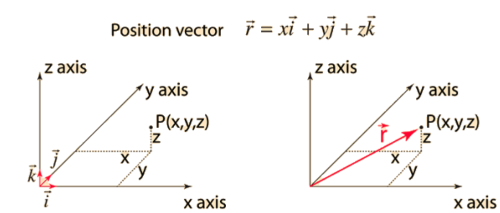
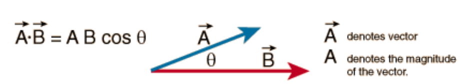
Syllabus: Scalar and vector fields : Gradient of a scalar function (use of del operator), Divergence and Curl product rules (explanation with geometrical representation), Line, surface and volume integrals (explanation with examples), Fundamental theorem for divergence and curl (statements only). **Electromagnetic waves** : Equation of Continuity, Displacement Current, Maxwell's equations in differential form (Derivation and physical significance), Derivation of wave equation (for one dimension), Velocity of em waves in free space and isotropic dielectric medium(derivation), Relation between refractive index and permittivity (qualitatively), Transverse nature of Plane em waves, , Poynting Vector, Energy density in electromagnetic field, Momentum and Pressure of em waves (derivation), Electromagnetic waves in a conducting medium – skin effect and skin depth

Scalar and vector fields: A scalar is an entity which has a magnitude. Eg. mass, time, distance, electric charge, electric potential, energy, temperature etc….
A vector is characterized by both magnitude and direction. Eg. displacement, velocity, acceleration, force, electric field, magnetic field etc….
A field is a quantity which can be specified everywhere in space as a function of position. The quantity that is specified may be a scalar or a vector.
A **scalar field** is one in which a scalar quantity is specified in magnitude at every point in a region of space as a function of position. For instance, temperature may be specified at every point in a room, then the room may be said to be in a region of temperature field which is a scalar field. In electromagnetism the scalar field is electric potential.
A **vector field** is one in which vector quantity is specified both in magnitude and direction at every point in a region of space as a function of position. For instance, consider the flow of water in a pipe. At every point in the region the water molecules have velocity given by $v(x, y, z)$. Thus water is said to be in the velocity field. In electromagnetism, electric fields and magnetic fields are the vector fields.

Basic vector concepts



A vector quantity called the position vector $\vec{r}$ in cartesian coordinate system can be represented as shown. In cartesian coordinates it is typical to use i, j and k to represent unit vectors in the x, y and z directions respectively. The position vector can be represented as $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

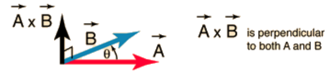


$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z \quad \text{where} \quad \vec{A} = A_x\vec{i} + A_y\vec{j} + A_z\vec{k}, \quad \vec{B} = B_x\vec{i} + B_y\vec{j} + B_z\vec{k}$$

The scalar product of two vectors can be constructed by taking the component of one vector in the direction of the other and multiplying it with the magnitude of the other vector. This can be expressed as shown above.

The vector product of two vectors can be constructed by taking the product of the magnitudes of the vectors times the sine of the angle (<180 degrees) between them. The magnitude of the vector product can be expressed as shown. The direction is given by the right-hand rule.

$$|\vec{A}\times\vec{B}|_{magnitude} = AB\sin\theta$$



$$\vec{A}\times\vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\vec{A}\times\vec{B} = \vec{i}(A_yB_z - A_zB_y) - \vec{j}(A_xB_z - A_zB_x) + \vec{k}(A_xB_y - A_yB_x)$$

Vector Identities : In the following identities, u and v are scalar functions while A and B are vector functions.

$$A\times(B\times C) = (C\times B)\times A = B(A\cdot C) - C(A\cdot B)$$
$$\nabla(uv) = u\nabla v + v\nabla u$$
$$\nabla(A\cdot B) = A\times(\nabla\times B) + (A\cdot\nabla)B + B\times(\nabla\times A) + (B\cdot\nabla)A$$
$$\nabla\cdot uA = u\nabla\cdot A + A\cdot\nabla u$$
$$\nabla\cdot(A\times B) = B\cdot\nabla\times A - A\cdot\nabla\times B$$
$$\nabla\times(uA) = u\nabla\times A - A\times\nabla u$$
$$\nabla\times(A\times B) = (B\cdot\nabla)A + A(\nabla\cdot B) - (A\cdot\nabla)B - B(\nabla\cdot A)$$
$$\nabla\times(\nabla\times A) = \nabla(\nabla\cdot A) - (\nabla\cdot\nabla)A$$

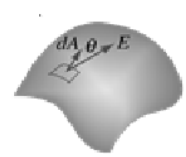
The collection of partial derivative operators is commonly called the del operator. $\nabla = \frac{\partial}{\partial x}i + \frac{\partial}{\partial y}j + \frac{\partial}{\partial z}k$

Gradient ∇f

Divergence $\nabla\cdot E$

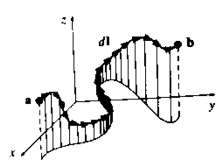
Curl $\nabla\times E$

Surface integral: A surface integral of a vector function E can be defined as the integral on a surface of the scalar product of E with area element dS. It is expressed in the following form $\int_s \vec{E}.\vec{dS}$. The direction of the area element is defined to be perpendicular to the area at that point on the surface.



The significance of this is understood in the example given. Let a surface encloses electric charges. To find the total electric flux (number of electric lines of force), the surface is divided into large number of infinitesimal areas. The flux through each such small area is found. The total flux is found by summation or integration. If the surface is a closed one then the integral is expressed as $\oint_s \vec{E}.\vec{dS} = \oint_s E\cos\theta\, ds$.

Line integral : A line integral of a vector function $\vec{A}$ can be defined as the integral over a curve which is the scalar product of A with line element *dl*. It is expressed in the following form $\int_c \vec{A}.\vec{dl}$. If the curve is a closed one then the integral is expressed as $\oint_c \vec{A}.\vec{dl}$. In the diagram shown, the path is divided in to large number of small lengths and the summation is taken over the entire path from a to b.

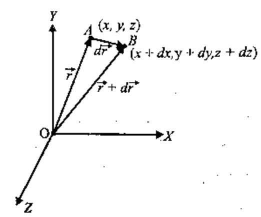


For example the line integral of electric field around a closed loop is equal to the voltage generated in that loop (Faraday's law) $\oint_c \vec{E}.\vec{dl} = -\frac{d\phi_B}{dt}$

Another example is the line integral of a force over a path is equal to the work done by that force on the path. $\int_c \vec{F}.\overrightarrow{dl} = W$.

Volume integral : A volume integral of a scalar function T can be defined as the integral over a volume which is the product of T with volume element dV. It is expressed in the following form $\int_v T\, dV$.
For example, if densityρ of a substance varies from point to point, then the volume integral gives the total mass. i.e. $\int_V \rho\, dV = m$.

Gradient of a scalar field : The gradient is a vector operation which operates on a scalar function to produce a vector. Its magnitude is the maximum rate of change of the function at the point of the gradient and which is pointed in the direction of that maximum rate of change.



Consider a scalar function f of coordinates $f(x, y, z)$ at a point A as shown. If it is given a small increment then its coordinates will be $f(x + dx, y + dy, z + dx)$ as shown.
Then the displacement $\overrightarrow{dr} = dx\,\hat{\imath} + dy\,\hat{\jmath} + dx\,\hat{k}$ ....(1)
The change in the scalar function f over a distance $\overrightarrow{dr}$ is given by
$df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial z}dz$ . .....(2)
We define a vector called ∇f called the gradient of scalar f. It is called grad .
As $\nabla = \frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial y}\hat{\jmath} + \frac{\partial}{\partial z}\hat{k}$ ....(3) thus $\nabla f = \frac{\partial f}{\partial x}\hat{\imath} + \frac{\partial f}{\partial y}\hat{\jmath} + \frac{\partial f}{\partial z}\hat{k}$ .....(4)
By taking product of (3) and (1) we get df.
Thus the change in scalar function $df = \nabla f.\overrightarrow{dr}$
$\mathrm{d}f = \left(\frac{\partial f}{\partial x}\hat{\imath} + \frac{\partial f}{\partial y}\hat{\jmath} + \frac{\partial f}{\partial z}\hat{k}\right).\left(dx\,\hat{\imath} + dy\,\hat{\jmath} + dx\,\hat{k}\right) = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial z}dz$

Relation between Electric field and potential

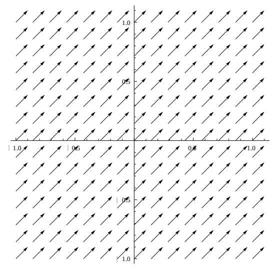
Electric potential is a scalar function. Consider a region in an electric field $\vec{E}$. Let the electric potential vary from one point to another in this field. (consider the diagram as shown above by considering f as V. Then the change in the electric potential in the Cartesian coordinate system is

$$dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz$$

We define a vector called ∇V called the gradient of electric potential V. It is called grad f given by
$\nabla V = \frac{\partial V}{\partial x}\hat{\imath} + \frac{\partial V}{\partial y}\hat{\jmath} + \frac{\partial V}{\partial z}\hat{k}$.
Thus $dV = \nabla V.\overrightarrow{dr}$ ....(1) where $dr = dx\,\hat{\imath} + dy\,\hat{\jmath} + dx\,\hat{k}$
By definition change in electric potential at a point is work done per unit charge given by
$dV = \frac{dW}{q_0} = -\frac{F.dr}{q_0} = -\vec{E}.\overrightarrow{dr}$ ....(2) (since electric field is force per unit charge)

Thus comparing (1) and (2) i.e. $dV = \nabla V.\overrightarrow{dr}$ and $dV = -\vec{E}.\overrightarrow{dr}$
we get $\nabla V.\overrightarrow{dr} = -\vec{E}.\overrightarrow{dr}$
or $\vec{\boldsymbol{E}} = -\boldsymbol{\nabla}\mathbf{V}$ or $\vec{\boldsymbol{E}} = -\boldsymbol{grad}\,\mathbf{V}$(Thus gradient of a scalar field is a vector)
Also electric field at a point is the negative potential gradient.

Divergence and curl of a vector



Two key concepts in vector calculus are **divergence** and **curl. T**he curl is also called circulation. Basically, **divergence** has to do with how a vector field changes its magnitude in the neighborhood of a point. The **curl** has to do with how vector field direction changes. Consider the graph as shown. It has both **zero divergence** and **zero curl**, because it doesn't change in either magnitude or direction in the neighborhood of any point.

Divergence of a vector

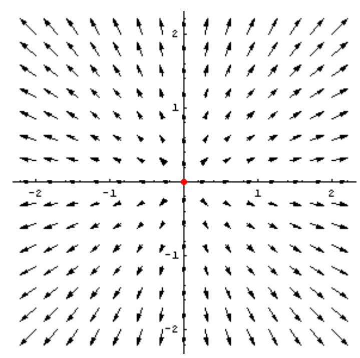
Divergence of a vector field $\vec{F}$ is a measure of net outward flux from a closed surface S enclosing a volume V, as the volume shrinks to zero.

$$div\vec{F} = \nabla.\vec{F} = \lim_{\Delta V \to 0} \frac{\oint_S \vec{F}.\overrightarrow{dS}}{\Delta V}.$$

where ΔV is the volume (enclosed by the closed surface S) in which the point P at which the divergence is being calculated is located. Since the volume shrinks to zero, the divergence is a point relationship and is a scalar.

As As $\nabla = \frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial y}\hat{\jmath} + \frac{\partial}{\partial z}\hat{k}$, and $F = F_x\hat{\imath} + F_y\hat{\jmath} + F_z\hat{k}$,

The divergence of vector is $\nabla.\mathrm{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$. This is a scalar.



Geometric representation: At the origin, all arrows point radially outward. Along any radius, the arrows are still pointing in the same direction, but they're getting longer. Since the length of an arrow in the plot is proportional to the strength of the field in the neighborhood of the arrow, we can see that as we go farther away from the origin along any radius, the direction of the field isn't changing, but its strength is. The graph shows a plot of the vector field F(x,y) If the arrows point outwards from the origin, then the divergence is positive and if it points inwards then divergence is negative.

Curl of a vector field

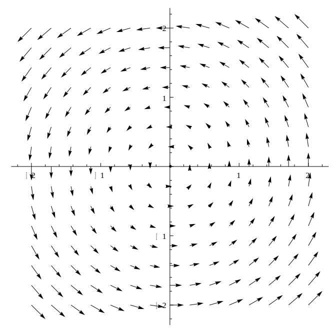
Curl of a vector is the measure of rotation of a field along a curve enclosing a surface. The curl of a vector is defined by the relation

$curl\,\vec{F} = \nabla \times \vec{F} = \lim_{\Delta S \to 0} \frac{\oint_c \vec{F}.\vec{dl}}{\Delta S}$.

The component of curl of a vector normal to the surface ΔS is equal to the line integral of the vector around the boundary of the surface divided by the area of the surface for limiting case in which surface area approaches zero. Curl of a vector is a vector. The direction of the resultant curl is along the axis of rotation.

Curl of a vector can be represented by the del operator as $\nabla \times \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$

or $\nabla \times \vec{F} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\hat{\imath} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\hat{\jmath} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\hat{k}$



Geometric representation: At the origin, the field is "circulating" around the point. Thus the name curl. At any distance from the origin, the arrows have constant length but constantly changing direction. Since their length is constant, the divergence is zero; since the direction is always changing, the **curl** is non-zero. The vectors all ***curl*** around a central point s shown in the diagram.

A field with **negative curl**, will simply curl in the opposite (clockwise) direction, The way to think about **curl** is a little more general than just vectors curling around a point. A good way to think about curl, is to think of a vector field as representing a fluid that actually flows and then to imagine a little paddlewheel placed at a point in the flow. If the paddlewheel will turn, then the field has a non-zero curl at that point. The field produce a net imbalance of forces and hence a **torque**, and thus the paddle will spin.

The divergence and curl are measures of how a vector field changes and thus they involve derivatives. In the study of electromagnetism, divergence and curl of electric and magnetic fields are used extensively to study the charges in the fields.

It is also important to note the following

$\nabla.(\nabla \times F) = 0$, $\nabla \times (\nabla f) = 0$ and $\nabla \times (\nabla \times F) = \nabla(\nabla.F) - \nabla^2 F$

Gradient theorem $\int_a^b (\nabla f).dl = f(b) - f(a)$

<u>**Divergence theorem :**</u> The divergence theorem states that the outward flux of a vector field through a closed surface is equal to the volume integral of the divergence over the region inside the surface.

It also means that the sum of all sources minus the sum of all sinks gives the net flow out of a region. Mathematically it is given by $\oint_S \vec{A}.\vec{dS} = \int_V (\nabla.\vec{A})\,dV$

Here $\vec{A}$ is a vector function. In a specific case of Gauss's law, the equation can be expressed as

$\oint_S \vec{D}.\overrightarrow{dS} = \int_V \left(\nabla . \vec{D}\right) dV$ where $\vec{D}$ is called electric displacement.

Stokes theorem : The stokes theorem states that the surface integral of the curl of a vector field taken over any surface is equal to the line integral of the field around the closed curve forming the periphery of the surface. Mathematically it can be represented as $\oint_C \vec{A}.\overrightarrow{dl} = \int_S \left(\nabla \times \vec{A}\right) dS$

Electromagnetism

For a very long time Electricity and Magnetism were developed and studied treating them as two independent fields. In 1820 Oersted observed that electric current produces magnetic fields and in 1831 Michael Faraday found that a moving magnet or a changing magnetic field generates electric current. The current is due to potential difference which in turn is due to electric fields. James Clerk Maxwell in 1865 formulated all the basic laws of electricity and magnetism into four equations called Maxwell's equations. These equations are the laws of electromagnetism. This Maxwell's theory unifies the three branches of physics, namely electricity, magnetism and optics into a single entity called electromagnetism. This justifies the Faraday's speculation that light is electrical in nature. Twenty years later in 1887, Henrich Hertz produced electromagnetic waves in the laboratory and called them as Maxwellian or radio waves.

Electric displacement

When a dielectric medium (insulator) is placed between the plates of a parallel plate capacitor, the medium gets polarised. Dipoles are induced in the dielectric and are oriented opposite to the electric field created by charges on the plates. This results in induced charges or bound charges opposite to the real charges. The electric dipole moment per unit volume is called the electric polarisation.

From Gauss' theorem $\oint_S \vec{E}.\overrightarrow{dS} = \frac{q}{\varepsilon_0}$.

In the absence of dielectric, above equation is $\overrightarrow{E_0}A = \frac{q}{\varepsilon_0}$ where $\oint_S \overrightarrow{dS} = A$

In the presence of dielectric $\vec{E}A = \frac{q-q'}{\varepsilon_0}$ where q' is the bound charges.

$\vec{E} = \frac{q-q'}{\varepsilon_0 A} = \frac{q}{\varepsilon_0 A} - \frac{q'}{\varepsilon_0 A}$. Thus $\frac{q}{\varepsilon_0 A} = \vec{E} + \frac{q'}{\varepsilon_0 A}$ or $\frac{q}{A} = \varepsilon_0\vec{E} + \frac{q'}{A}$

Thus $\boldsymbol{\vec{D} = \varepsilon_0\vec{E} + P}$ where $\frac{q}{A} = \vec{D}$ and $\frac{q'}{A} = P$

Here D is called the electric displacement and P is the dielectric polarization.

$\vec{D}$ depends only on the free charges and $\vec{E}$ depends on both free and bound charges.

It can be shown that $\vec{D} = \varepsilon_0 \varepsilon_r \vec{E} = \varepsilon \vec{E}$ and $\vec{D} = \varepsilon_0 \vec{E}$ in free space as $\varepsilon_r = 1$.

Equation of continuity

The principle of conservation of charge states as – If the net charge crossing a surface bounded by a volume is not zero, then the charge density within the volume must change with time in a manner that the time rate of decrease of charge within the volume is equal to the net rate of flow of charge out of the volume. This is expressed by the equation of continuity.

Consider a closed surface S enclosing a volume V of electric charge distribution. The net amount of charge crossing unit area of a surface normal to the direction of charge flow in unit time is defined as the current density $\vec{J}$. The total electric current I flowing outward through a closed surface is given by $I = \oint_S \vec{J}.\overrightarrow{dS}$ ….(1)

As the charge is conserved , it should decrease within the volume at the rate

$I = -\frac{dq}{dt}$ ….(2) As $q = \int_V \rho\, dV$ thus $I = -\frac{d}{dt}\int_V \rho\, dV$ ….(3)

ρ is the volume charge density. Comparing equations (1) and (3)

$\oint_S \vec{J}.\overrightarrow{dS} = -\frac{d}{dt}\int_V \rho\, dV$ or $\oint_S \vec{J}.\overrightarrow{dS} = -\int_V \frac{d\rho}{dt}\, dV$ ….(4)

From Guass' divergence theorem $\oint_S \vec{J}.\overrightarrow{dS} = \int_V \left(\nabla.\vec{J}\right) dV$ ….(5)

Substituting (4) in (5) $\int_V \left(\nabla.\vec{J}\right) dV = -\int_V \frac{d\rho}{dt}\, dV$

For any arbitrary volume, the above equation becomes $\nabla.\vec{J} = -\frac{d\rho}{dt}$

or $\boldsymbol{\nabla.\vec{J} + \frac{d\rho}{dt} = 0}$ …(6) This is called the equation of continuity which is the mathematical form of law of conservation of charge. For stationary current or steady current, $\frac{d\rho}{dt} = 0$. Thus $\nabla.\vec{J} = 0$.

Maxwell's equations

Maxwell using the four basic laws of electricity and magnetism, namely Gauss's law, Biot Savart's law, Faraday's law andAmpere's circuital law, formulated four fundamental equations called Maxwell's equations. The four Maxwell's equations are

1. $\nabla.\vec{D} = \rho$ or $div\,\vec{D} = \rho$
2. $\nabla.\vec{B} = 0$ or $div\,\vec{B} = 0$
3. $\nabla\times\vec{E} = -\frac{\partial B}{\partial t}$ or $curl\,\vec{E} = -\frac{\partial\vec{B}}{\partial t}$
4. $\nabla\times\vec{B} = \mu_0\left(\vec{J} + \frac{\partial\vec{D}}{\partial t}\right)$ or $curl\,\vec{B} = \mu_0\left(\vec{J} + \frac{\partial\vec{D}}{\partial t}\right)$

1 Derivation of $\boldsymbol{\nabla.\vec{D} = \rho}$

Consider a surface S which encloses a volume V in a dielectric medium. The total charge in the dielectric medium consists of free charges and the polarization charges. If $\rho\ and\ \rho_P$ are the free charge density and the polarization charge density respectively at a point in a small volume element. Then according to Gauss's law

$\oint_S \vec{E}.\overrightarrow{dS} = \frac{q}{\varepsilon_0}$ .....(1) where $\vec{E}$ is the electric field due to all charges and ε_0 is the permittivity of free space. Also $q = \int_V (\rho + \rho_P)\, dV$, equation (1) becomes

$\oint_S \vec{E}.\overrightarrow{dS} = \frac{1}{\varepsilon_0}\int_V (\rho + \rho_P)\, dV$ ....(2)

[Also $\int_V \rho_P\, dV = \oint_S \vec{P}.\overrightarrow{dS}$. This is because $q_P = \int_V \rho_P\, dV\ and\ P = \frac{q_P}{dS}$

Using divergence theorem $\oint_S \vec{P}.\overrightarrow{dS} = -\int_V \nabla.\mathrm{P}\, dV$.

Thus $\int_V \rho_P\, dV = -\int_V \nabla.\mathrm{P}\, dV$. For any arbitrary volume $\rho_P = -\nabla.\vec{P}$

Putting this in equation (2) $\oint_S \vec{E}.\overrightarrow{dS} = \frac{1}{\varepsilon_0}\int_V \left(\rho - \nabla.\vec{P}\right) dV$ ....(3)

Again from the divergence theorem $\oint_S \vec{E}.\overrightarrow{dS} = \int_V \nabla.\vec{E}\ dV$ .....(4)

Comparing (3) and (4) $\int_V \nabla.\vec{E}\ dV = \frac{1}{\varepsilon_0}\int_V \left(\rho - \nabla.\vec{P}\right) dV$

Rearranging the above equation, $\int_V \nabla.\varepsilon_0\vec{E}\ dV = \int_V \left(\rho - \nabla.\vec{P}\right) dV$

Or $\int_V \left(\nabla.\varepsilon_0\vec{E} + \nabla.\vec{P}\right) dV = \int_V \rho\, dV$ or $\int_V \nabla.\left(\varepsilon_0\vec{E} + \vec{P}\right) dV = \int_V \rho\, dV$

As $\varepsilon_0\vec{E} + \vec{P} = \vec{D}$ called electric displacement, the above equation is

$\int_V \nabla.\vec{D}\ dV = \int_V \rho\, dV$. This is true for all volumes. Thus the integrand vanishes.

Thus $\boldsymbol{\nabla.\vec{D} = \rho}$ or $\boldsymbol{div\,\vec{D} = \rho}$

2. Derivation of $\boldsymbol{\nabla.\vec{B} = 0}$

According to Biot - Savart's law, the magnetic field $\vec{B}$ at a point due to a current element $I\overrightarrow{dl}$ is given by $\vec{B} = \frac{\mu_0}{4\pi}\int I\left(\overrightarrow{dl} \times \frac{\vec{r}}{r^3}\right)$....(1)

Where $\vec{r}$ is the position vector drawn from the current carrying element $\overrightarrow{dl}$ to the point where the magnetic field is measured. μ_0 is called the permeability of free space. Taking divergence on both sides of equation (1), we get

$\nabla.\vec{B} = \frac{\mu_0}{4\pi} I \int \nabla.\left(\overrightarrow{dl} \times \frac{\vec{r}}{r^3}\right)$ ....(2)

Using the vector identity $\nabla.\left(\vec{A} \times \vec{B}\right) = \vec{B}.\left(\nabla \times \vec{A}\right) - \vec{A}.\left(\nabla \times \vec{B}\right)$

we have $\nabla.\left(\overrightarrow{dl} \times \frac{\vec{r}}{r^3}\right) = \frac{\vec{r}}{r^3}.\left(\nabla \times \overrightarrow{dl}\right) - \overrightarrow{dl}.\left(\nabla \times \frac{\vec{r}}{r^3}\right)$ .....(3)

since $\overrightarrow{dl}$ is not the function of coordinates of the field point, $\nabla \times \overrightarrow{dl} = 0$

Thus equation (3) is $\nabla.\left(\overrightarrow{dl} \times \frac{\vec{r}}{r^3}\right) = \overrightarrow{dl}.\left(\nabla \times \frac{\vec{r}}{r^3}\right)$ ...(3)

Also $\nabla \times \frac{\vec{r}}{r^3} = \nabla \times \nabla\left(\frac{1}{r}\right)$ This is because $\frac{\vec{r}}{r^3} = -\nabla\left(\frac{1}{r}\right) = -\frac{d}{dr}\left(\frac{1}{r}\right)$

But $\nabla \times \nabla\left(\frac{1}{r}\right) = 0$ (curl of gradient of a function is zero). Therefore $\nabla \times \frac{\vec{r}}{r^3} = 0$

Thus equation (3) is $\nabla.\left(\overrightarrow{dl} \times \frac{\vec{r}}{r^3}\right) = 0$

Thus in equation (2) RHS is zero. Thus $\boldsymbol{\nabla.\vec{B} = 0}$ or $\boldsymbol{div\,\vec{B} = 0}$

3. Derivation of $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

According to the Faraday's law, the induced emf in a closed loop or a circuit is given by$e = -\frac{\partial \phi}{\partial t}$ ....(1)

where ϕ is the magnetic flux linked with the circuit, given by

$\phi = \int_S \vec{B}.\overrightarrow{dS}$ ...(2)

Substituting for ϕ from (2) in (1), $e = -\frac{\partial}{\partial t}\int_S \vec{B}.\overrightarrow{dS}$

Or $e = -\int_S \frac{\partial \vec{B}}{\partial t}.\overrightarrow{dS}$ .....(3)

The induced emf is also defined as the amount of work done in taking a unit charge once round a closed path, i.e. $e = \oint_C \vec{E}.\overrightarrow{dl}$ ....(4)

Where $\vec{E}$ is the electric field intensity at the element $\overrightarrow{dl}$ of the loop.

According to stoke's theorem $\oint_C \vec{E}.\overrightarrow{dl} = \int_S \left(\nabla \times \vec{E}\right) dS$ ...(5)

Comparing (4) and (5) $e = \int_S \left(\nabla \times \vec{E}\right) dS$ ....(6)

Comparing (3) and (6) we get, $\int_S \left(\nabla \times \vec{E}\right) dS = -\int_S \frac{\partial \vec{B}}{\partial t}.\overrightarrow{dS}$

Thus for an arbitrary volume, the integrand vanishes.

Thus $\boldsymbol{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$ or $\boldsymbol{curl\,\vec{E} = -\frac{\partial \vec{B}}{\partial t}}$

4. Derivation of $\nabla \times \vec{B} = \mu_0\left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)$

According to Ampere's circuital law, work done in taking a unit magnetic pole once round a closed arbitrary path linked with the current is given by $\oint_C \vec{B}.\overrightarrow{dl} = \mu_0 I$ ...(1)

where $\vec{B}$ is the magnetic field. But the total current enclosed by a surface S is

$I = \int_S \vec{J}.\overrightarrow{dS}$ ....(2) where $\vec{J}$ is the current density.

From the above two equations we get $\oint_C \vec{B}.\overrightarrow{dl} = \mu_0 \int_S \vec{J}.\overrightarrow{dS}$ ....(3)

According to Stoke's theorem $\oint_C \vec{B}.\overrightarrow{dl} = \int_S \left(\nabla \times \vec{B}\right) dS$ ....(4)

Comparing (3) and (4) $\int_S \left(\nabla \times \vec{B}\right) dS = \mu_0 \int_S \vec{J}.\overrightarrow{dS}$

For an arbitrary surface, $\nabla \times \vec{B} = \mu_0 \vec{J}$ ....(5)
This equation is valid for only steady closed current and is found to be inconsistent with the equation of continuity as explained below.
Taking divergence on both sides of (5) $\nabla.(\nabla \times \vec{B}) = \mu_0(\nabla.\vec{J})$
But $\nabla.(\nabla \times \vec{B}) = 0$. Thus $\nabla.\vec{J} = 0$
According to the equation of continuity $\nabla.\vec{J} = -\frac{\partial \rho}{\partial t}$. Thus equation (5) is inconsistent with the law of conservation of charge or the equation of continuity. This inconsistency in Ampere's law was recognized by Maxwell. He said that along with the term current density $\vec{J}$ in equation (5) there must be another term called $\vec{J_d}$ that should be added to $\vec{J}$.
Now the equation (5) becomes $\nabla \times \vec{B} = \mu_0(\vec{J} + \vec{J_d})$ ....(6)
Taking divergence on both sides of the above equation,

$$\nabla.\,(\nabla \times \vec{B}) = \mu_0 \nabla.(\vec{J} + \vec{J_d})$$

As $\nabla.(\nabla \times \vec{B}) = 0$, the above equation is $0 = \mu_0 \nabla.(\vec{J} + \vec{J_d})$
or $0 = \mu_0 \nabla.\vec{J} + \mu_0 \nabla.\vec{J_d}$ or $\nabla.\vec{J} = -\nabla.\vec{J_d}$ ......(7)
From equation of continuity $\nabla.\vec{J} = -\frac{\partial \rho}{\partial t}$ .....(8)
Comparing (7) and (8) $\nabla.\vec{J_d} = \frac{\partial \rho}{\partial t}$ ....(9)
From Maxwell's first equation, $\nabla.\vec{D} = \rho$
Thus equation (9) is $\nabla.\vec{J_d} = \frac{\partial}{\partial t}(\nabla.\vec{D})$ or $\nabla.\vec{J_d} = \nabla.\frac{\partial \vec{D}}{\partial t}$
or $\vec{J_d} = \frac{\partial \vec{D}}{\partial t}$ ....(10) where $\vec{D}$ is the electric displacement.
Substituting for $\vec{J_d}$ from (10) in (6), we get $\boldsymbol{\nabla \times \vec{B} = \mu_0 \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)}$
or $\boldsymbol{curl\,\vec{B} = \mu_0 \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)}$
This equation can also be expressed as $\boldsymbol{\nabla \times \vec{B} = \mu_0 \left(\vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right)}$
This modified equation is consistent with conservation of charge. It is applicable to both steady and varying currents. The term $\vec{J_d} = \frac{\partial \vec{D}}{\partial t}$ is called the displacement current density analogous to conduction current$\vec{J}$.

Characteristics of displacement current $\boldsymbol{\vec{J_d} = \frac{\partial \vec{D}}{\partial t}}$

1. It is that current that comes into existence (in addition to conduction current) whenever the electric field and hence the electric flux changes with time.
2. It only adds to current density in Ampere's circuital law. As it produces magnetic field so it is called a current.

3. The magnitude of displacement current is equal to the rate of displacement of charge from one plate to other plate of a capacitor.
4. Together with the conduction current, displacement current satisfies the property of continuity.
5 The displacement current density is as real as the conduction current density since both produce magnetic fields. The sum of these two is called the total current density.
6 Displacement current proves that the electric field vector $\vec{E}$ and the magnetic field vector $\vec{B}$ are interlinked. i.e. the time rate of change of electric field $\vec{E}$ induces magnetic field $\vec{B}$ and vice-versa.

Physical significance of Maxwell's equations

1 Consider the first Maxwell equation $\nabla . \vec{D} = \rho$

Integrating over an arbitrary volume $\int_V \nabla . \vec{D}\ dV = \int_V \rho\, dV$

Applying divergence theorem to LHS $\int_V \vec{D}\, dS = \int_V \rho\, dV$ or $\int_V \vec{D}\, dS = q$ ....(1)
where S is the surface enclosing volume V and q is the net charge contained in the volume V. The equation signifies that (i) net outward flux of electric displacement vector through the surface enclosing a volume is equal to the net charge contained within that volume and (ii) either the lines of flux enter or leave the closed surface. They do not form closed loops. This suggests that there are sources due to positive charges and sinks due to negative charges.

2. Consider the second Maxwell's equation $\nabla . \vec{B} = 0$

Integrating over an arbitrary volume $\int_V \vec{B}\, dV = 0$

Applying Gauss divergence theorem $\int_S \vec{B}.d\vec{S} = 0$ ...(2) where S is the surface enclosing the volume V. This equation signifies that net outward flux of magnetic induction $\vec{B}$ through any closed surface is equal to zero. i.e. magnetic flux lines are continuous closed loops implying that there are no sources or sinks in magnetic fields. Thus there are no magnetic monopoles and they exist as dipoles.

3. Consider the third Maxwell's equation $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

Integrating over a surface S bounded by a curve $\int_S \left(\nabla \times \vec{E}\right) dS = -\int_S \frac{\overrightarrow{\partial B}}{\partial t}.\overrightarrow{dS}$

Applying Stoke's theorem to LHS $\int_C \vec{E}.\overrightarrow{dl} = -\int_S \frac{\overrightarrow{\partial B}}{\partial t}.\overrightarrow{dS}$ .....(3)

The LHS of the above equation is called electromotive force. Thus the equation signifies that the electromotive force around a closed path is equal to the negative rate of change of magnetic flux linked with the path.

4. Consider the fourth Maxwell's equation $\nabla \times \vec{B} = \mu_0 \left(\vec{j} + \frac{\partial \vec{D}}{\partial t}\right)$

Integrating over a surface S bounded by a curve C , $\int_S \nabla \times \vec{B} = \mu_0 \int_S \left(\vec{j} + \frac{\partial \vec{D}}{\partial t}\right)$

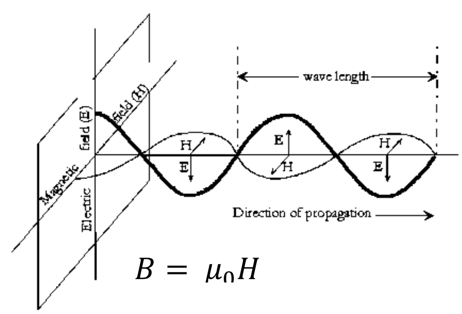
Applying Stoke's theorem to LHS, $\int_C \vec{B}.\vec{dl} = \mu_0 \int_S \left(\vec{j} + \frac{\partial \vec{D}}{\partial t}\right)$ ....(4)

The LHS of the above equation is called the magnetomotive force. Thus the magnetomotive force around a closed path is equal to the sum of the conduction current and the displacement current through any surface bounded by the path. The equations (1), (2), (3) and (4) are called the integral forms of Maxwell's equations.

Electromagnetic waves :

An electromagnetic wave exists when the changing magnetic field causes a changing electric field, which then causes another changing magnetic field, and so on forever. Unlike a static field, a wave cannot exist unless it is moving. Once created, an electromagnetic wave will continue on forever unless it is absorbed by matter. Electromagnetic radiation is created when an atomic particle, such as an electron, is accelerated by an electric field, causing it to move. The movement produces oscillating electric and magnetic fields, which travel at right angles to each other in a bundle of light energy called a photon.

Electromagnetic wave equation : Consider a homogeneous medium (a medium in which ε, μ *and* σ remain constant throughout the medium) and an isotropic medium (a medium is isotropic in which ε is a scalar constant so that $\vec{D}$ *and* $\vec{E}$ have same direction everywhere) free from charges and currents. Thus $\rho = 0$ and current density $\vec{J} = \sigma\vec{E} = 0$ $(as\ \sigma = 0)$ Here σ is the conductivity of the medium. Consider medium to be free space so that $\varepsilon = \varepsilon_0$ *and* $\mu = \mu_0$. Also $\vec{D} = \varepsilon_0\vec{E}$.



$B = \mu_0 H$

The four Maxwell's equations are

$\nabla.\vec{D} = \rho$, $\nabla.\vec{B} = 0$, $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}\nabla \times \vec{B} = \mu_0\left(\vec{j} + \frac{\partial \vec{D}}{\partial t}\right)$

By applying the above conditions to Maxwell's equations, we get

$\nabla.\vec{E} = 0$ .... (1) $\nabla.\vec{B} = 0$ ......(2)

$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ ……(3) $\nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$ …..(4)

Taking curl on both sides of equation (3) $\nabla \times \nabla \times \vec{E} = -\nabla \times \frac{\partial \vec{B}}{\partial t}$

or $\nabla \times (\nabla \times \vec{E}) = -\frac{\partial (\nabla \times \vec{B})}{\partial t}$ ……(5)

substituting for $\nabla \times \vec{B}$ from (4) in (5), we get $\nabla \times (\nabla \times \vec{E}) = -\frac{\partial}{\partial t}\left(\mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right)$

or $\nabla \times (\nabla \times \vec{E}) = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$ ……(6)

But $\nabla \times (\nabla \times \vec{E}) = \nabla(\nabla . \vec{E}) - \nabla^2 \vec{E}$

Thus equation (6) is $\nabla(\nabla . \vec{E}) - \nabla^2 \vec{E} = = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$

From equation (1) $\nabla . \vec{E} = 0$, thus the above equation becomes

$-\nabla^2 \vec{E} = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$ or $\boldsymbol{\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}}$ ….(7)

Repeating the above process by taking the curl on both sides of equation (4),

$\nabla \times (\nabla \times \vec{B}) = -\nabla \times \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$ or $\nabla \times (\nabla \times \vec{B}) = -\mu_0 \varepsilon_0 \frac{\partial (\nabla \times \vec{E})}{\partial t}$

substituting for $\nabla \times \vec{E}$ from (3) the above equation, $\nabla \times (\nabla \times \vec{B}) = -\frac{\partial}{\partial t}\left(\mu_0 \varepsilon_0 \frac{\partial \vec{B}}{\partial t}\right)$

But $\nabla \times (\nabla \times \vec{B}) = \nabla(\nabla . \vec{B}) - \nabla^2 \vec{B}$

Thus we have $\nabla(\nabla . \vec{B}) - \nabla^2 \vec{B} = = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$

From equation (2) $\nabla . \vec{B} = 0$, thus the above equation becomes

$\boldsymbol{\nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}}$ ….(8)

Equations (7) and (8) are called the electromagnetic wave equations.

Velocity of electromagnetic waves in free space

The standard wave equation is given by $\nabla^2 \vec{\psi} = \frac{1}{v^2} \frac{\partial^2 \vec{\psi}}{\partial t^2}$

The electromagnetic wave equations are $\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$ and $\nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$

Comparing the electromagnetic wave equations with the standard equation, we find

$\frac{1}{v^2} = \mu_0 \varepsilon_0$ or $v^2 = \frac{1}{\mu_0 \varepsilon_0}$ or $\boldsymbol{v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}}$

The above expression gives the velocity of an electromagnetic wave in free space As $\mu_0 = 4\pi \times 10^{-7}\ Hm^{-1}$ and $\varepsilon_0 = 8.854 \times 10^{-12}\ Fm^{-1}$, thus the value of velocity is $v = \frac{1}{\sqrt{4\pi \times 10^{-7} \times 8.854 \times 10^{-12}}} = 2.9979 \times 10^8\ ms^{-1}$

Therefore the velocity of electromagnetic wave is equal to $2.9979 \times 10^8\ ms^{-1}$ which is exactly the velocity of light wave, i.e. electromagnetic waves travel with the velocity of light. Thus light is a electromagnetic wave and the velocity of electromagnetic wave in free space is given by $\boldsymbol{c} = \frac{\mathbf{1}}{\sqrt{\boldsymbol{\mu_0 \varepsilon_0}}}$. Also $c = \frac{E}{B}$

Electromagnetic wave equation in a diectric medium

Consider a homogeneous and an isotropic medium free from charges and currents. Thus $\rho = 0$ and current density $\vec{J} = \sigma \vec{E} = 0$ $(as\ \sigma = 0)$ Here σ is the conductivity of the medium. Consider medium to be dielectric with absolute permittivity as ε and absolute permeability as μThus the electric displacement is $\vec{D} = \varepsilon\, \vec{E}$.

The four Maxwell's equations are

$\nabla . \vec{D} = \rho$, $\nabla . \vec{B} = 0$, $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \nabla \times \vec{B} = \mu \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right)$

By applying the above conditions to Maxwell's equations, we get

$\nabla . \vec{E} = 0$ .... (1) $\nabla . \vec{B} = 0$ ......(2)

$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ ......(3) $\nabla \times \vec{B} = \mu\, \varepsilon \frac{\partial \vec{E}}{\partial t}$ .....(4)

Taking curl on both sides of equation (3) $\nabla \times \nabla \times \vec{E} = -\nabla \times \frac{\partial \vec{B}}{\partial t}$

or $\nabla \times \left(\nabla \times \vec{E} \right) = -\frac{\partial (\nabla \times \vec{B})}{\partial t}$ ......(5)

substituting for $\nabla \times \vec{B}$ from (4) in (5), we get $\nabla \times \left(\nabla \times \vec{E} \right) = -\frac{\partial}{\partial t} \left(\mu\, \varepsilon \frac{\partial \vec{E}}{\partial t} \right)$

or $\nabla \times \left(\nabla \times \vec{E} \right) = -\mu\, \varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$ ......(6)

But $\nabla \times \left(\nabla \times \vec{E} \right) = \nabla \left(\nabla .\ \vec{E} \right) - \nabla^2 \vec{E}$

Thus equation (6) is $\nabla \left(\nabla .\ \vec{E} \right) - \nabla^2 \vec{E} = = -\mu\, \varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$

From equation (1) $\nabla .\ \vec{E} = 0$, thus the above equation becomes $-\nabla^2 \vec{E} = -\mu\, \varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$

or $\boldsymbol{\nabla^2 \vec{E} = \mu\, \varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}}$ ....(7)

Repeating the above process by taking the curl on both sides of equation (4),

$\nabla \times \left(\nabla \times \vec{B} \right) = -\nabla \times \mu\, \varepsilon \frac{\partial \vec{E}}{\partial t}$ or $\nabla \times \left(\nabla \times \vec{B} \right) = -\mu\, \varepsilon \frac{\partial (\nabla \times \vec{E})}{\partial t}$

substituting for $\nabla \times \vec{E}$ from (3) the above equation, $\nabla \times \left(\nabla \times \vec{B} \right) = -\frac{\partial}{\partial t} \left(\mu\, \varepsilon \frac{\partial \vec{B}}{\partial t} \right)$

But $\nabla \times \left(\nabla \times \vec{B} \right) = \nabla \left(\nabla .\ \vec{B} \right) - \nabla^2 \vec{B}$

Thus we have $\nabla \left(\nabla .\ \vec{B} \right) - \nabla^2 \vec{B} = = -\mu\, \varepsilon \frac{\partial^2 \vec{B}}{\partial t^2}$

From equation (2) $\nabla .\ \vec{B} = 0$, thus the above equation becomes

$\boldsymbol{\nabla^2 \vec{B} = \mu\, \varepsilon \frac{\partial^2 \vec{B}}{\partial t^2}}$ ....(8)

Equations (7) and (8) are called the electromagnetic wave equations.

Velocity of electromagnetic waves in the dielctric medium

The standard wave equation is given by $\nabla^2\vec{\psi} = \frac{1}{v^2}\frac{\partial^2\vec{\psi}}{\partial t^2}$

The electromagnetic wave equations are $\nabla^2\vec{E} = \mu\,\varepsilon\frac{\partial^2\vec{E}}{\partial t^2}$ and $\nabla^2\vec{B} = \mu\,\varepsilon\frac{\partial^2\vec{B}}{\partial t^2}$

Comparing the electromagnetic wave equations with the standard equation, we find

$\frac{1}{v^2} = \mu\,\varepsilon$ or $v^2 = \frac{1}{\mu\,\varepsilon}$ or $\boldsymbol{v} = \frac{\mathbf{1}}{\sqrt{\boldsymbol{\mu\,\varepsilon}}}$

Relation between refractive index and permittivity

The velocity of electromagnetic wave in a dielectric medium is given by

$v = \frac{1}{\sqrt{\mu\varepsilon}}$ ....(1)

where μ called the absolute permeability given by $\mu = \mu_0\mu_r$ and ε is called the absolute permittivity given by $\varepsilon = \varepsilon_0\varepsilon_r$. Also μ_r and ε_r are called relative permeability and relative permittivity of the medium respectively. Thus

$v = \frac{1}{\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}} = \frac{1}{\sqrt{\mu_0\varepsilon_0\mu_r\varepsilon_r}}$.

As $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$, thus $v = \frac{c}{\sqrt{\mu_r\varepsilon_r}}$ ...(2)

The refractive index of the medium is given by $n = \frac{c}{v}$ ....(3)

Substituting for v from (2) in (3), we have $n = \sqrt{\mu_r\varepsilon_r}$

For a non – magnetic medium, $\mu_r = 1$. Thus $\boldsymbol{n} = \sqrt{\boldsymbol{\varepsilon_r}}$

To show that electromagnetic waves are transverse in nature

Consider the electromagnetic wave equations $\nabla^2\vec{E} = \mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}$ and $\nabla^2\vec{B} = \mu_0\varepsilon_0\frac{\partial^2\vec{B}}{\partial t^2}$

The solutions of these equations respectively are

$\vec{E} = \overrightarrow{E_0}e^{-i(\omega t-\vec{k}.\vec{r})}$ ...(1) and $\vec{B} = \overrightarrow{B_0}e^{-i(\omega t-\vec{k}.\vec{r})}$ .....(2)

where $\vec{k}$ is called the propagation constant and its magnitude is $k = \frac{2\pi}{\lambda}$ and ω is the angular frequency.

Differentiating equation (1) w.r.t. space $\nabla.\vec{E} = i\vec{k}\overrightarrow{E_0}e^{-i(\omega t-\vec{k}.\vec{r})}$ or $\nabla.\vec{E} = i\vec{k}\vec{E}$

Thus $\nabla = i\vec{k}$ .....(3)

Differentiating (1) w.r.t. time, we have $\frac{\partial\vec{E}}{\partial t} = -i\omega\,\overrightarrow{E_0}e^{-i(\omega t-\vec{k}.\vec{r})}$ or $\frac{\partial\vec{E}}{\partial t} = -i\omega\,\vec{E}$

Thus $\frac{\partial}{\partial t} = -i\omega$ ....(4)

Substituting these in Maxwell's equations in free space

1. $\nabla.\vec{E} = 0$ i.e. $i\vec{k}.\vec{E} = 0$ This shows $\vec{k}$ is perpendicular to electric field vector $\vec{E}$

2. $\nabla . \vec{B} = 0$, $i\vec{k} . \vec{B} = 0$ This shows $\vec{k}$ is perpendicular to magnetic field vector $\vec{B}$
3. $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ i.e. $i\vec{k} \times \vec{E} = -(-i\omega)\vec{B}$ or $i\vec{k} \times \vec{E} = i\omega\vec{B}$ This shows that $\vec{B}$ is perpendicular to both $\vec{k}$ and $\vec{E}$
4. $\nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$ i.e. $i\vec{k} \times \vec{B} = \mu_0\varepsilon_0(-i\omega)\vec{E}$ or $i\vec{k} \times \vec{B} = -i\omega\mu_0\varepsilon_0\vec{E}$ This shows that $\vec{E}$ is perpendicular to both $\vec{k}$ and $\vec{B}$

(In the first two cases, the dot product of two vectors is zero means the vectors are perpendicular. In the second two cases the cross product is not zero means the two vectors are perpendicular) $\vec{k}$ indicates the direction in which wave propagates. From the above four cases it is seen that all the three vectors $\vec{k}, \vec{E}$ *and* $\vec{B}$ are mutually perpendicular to each other. This shows that electromagnetic waves are transverse in nature.

Relation between electric and magnetic vector

Consider the the Maxwell's equation in free space $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

As $\nabla = i\vec{k}$ and $\frac{\partial}{\partial t} = -i\omega$, above equation is $i\vec{k} \times \vec{E} = i\omega\vec{B}$ or $\vec{k} \times \vec{E} = \omega\vec{B}$ ....(1)

As $\vec{k} = \frac{2\pi}{\lambda}$, multiplying and dividing this equation by f i.e. frequency we have $k = \frac{2\pi f}{\lambda f} = \frac{\omega}{c}$. $[\because \omega = 2\pi f$ and $c = f\lambda]$

Thus $\vec{k} = \frac{\omega}{c}\hat{n}$ .....(2) where $\hat{n}$ is normal unit vector perpendicular to both $\vec{E}$ *and* $\vec{B}$.

Substituting for $\vec{k}$ from (2) in (1) $\frac{\omega}{c}\hat{n} \times \vec{E} = \omega\vec{B}$ or $\hat{n} \times \vec{E} = c\vec{B}$

or $\frac{\hat{n} \times \vec{E}}{\vec{B}} = c$ or $\left|\frac{\vec{E}}{\vec{B}}\right| = c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$

Thus $\boldsymbol{E} = \boldsymbol{c}\vec{\boldsymbol{B}}$ is the relation between electric field vector and the magnetic field vector in vacuum. Thus $\vec{E}$ and $\vec{B}$ are in phase in vacuum and change simultaneously. The above relation is also called phase relation between $\vec{E}$ and $\vec{B}$.

Similarly in a material medium or a dielectric medium, $\left|\frac{\vec{E}}{\vec{B}}\right| = v = \frac{1}{\sqrt{\mu\varepsilon}}$ or $\vec{\boldsymbol{E}} = \boldsymbol{v}\vec{\boldsymbol{B}}$.

Energy carried by electromagnetic waves – Poynting theorem

Consider some charge and current in a region producing electric and magnetic fields. The work done on the charges q by the electromagnetic forces in an interval of time dt through a distance dl is given by $dW = \vec{F}.\overrightarrow{dl}$ where $\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$ and $\overrightarrow{dl} = \vec{v}\,dt$

Thus $dW = q(\vec{E} + \vec{v}\times\vec{B}).\vec{v}\,dt = q\,\vec{E}.\vec{v}\,dt$ $\quad (\because\ \vec{v}\times\vec{v} = 0)$

As $q = \rho\, dV$ and $\rho\,\vec{v} = \vec{J}$ where $\vec{J}$ is surface current density and ρ volume charge density, we have $\frac{dW}{dt} = \vec{E}.\vec{J}\,dV$.

In integral form, the rate at which work is done on all charges in a volume V is $\frac{dW}{dt} = \int_V (\vec{E}.\vec{J})dV$ ....(1) Thus $\vec{E}.\vec{J}$ represents work done per unit time per unit volume or power delivered per unit volume.

From Maxwell equation $\nabla\times\vec{B} = \mu_0\left(\vec{J} + \frac{\partial\vec{D}}{\partial t}\right)$, thus

$\vec{J} = \frac{1}{\mu_0}(\nabla\times\vec{B}) - \varepsilon_0\frac{\partial\vec{E}}{\partial t}$ $\quad (\because\ \vec{D} = \varepsilon_0\vec{E})$

Now we have $\vec{E}.\vec{J} = \frac{1}{\mu_0}\vec{E}.(\nabla\times\vec{B}) - \varepsilon_0\vec{E}.\frac{\partial\vec{E}}{\partial t}$

From the product rule $\nabla.(\vec{E}\times\vec{B}) = \vec{B}.(\nabla\times\vec{E}) - \vec{E}.(\nabla\times\vec{B})$ ....(2)

From Maxwell's equation $\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

Equation (2) can be rewritten as $\vec{E}.(\nabla\times\vec{B}) = -\vec{B}.\frac{\partial\vec{B}}{\partial t} - \nabla.(\vec{E}\times\vec{B})$ ....(3)

Also $\vec{B}.\frac{\partial\vec{B}}{\partial t} = \frac{1}{2}\frac{\partial}{\partial t}(B^2)$ and $\vec{E}.\frac{\partial\vec{E}}{\partial t} = \frac{1}{2}\frac{\partial}{\partial t}(E^2)$ ....(4)

(3) and (4) in (2) gives $\vec{E}.\vec{J} = \frac{1}{\mu_0}\left[-\frac{1}{2}\frac{\partial}{\partial t}(B^2) - \nabla.(\vec{E}\times\vec{B})\right] - \varepsilon_0\frac{1}{2}\frac{\partial}{\partial t}(E^2)$

or $\vec{E}.\vec{J} = -\frac{1}{2}\frac{\partial}{\partial t}\left(\varepsilon_0E^2 + \frac{1}{\mu_0}B^2\right) - \frac{1}{\mu_0}\nabla.(\vec{E}\times\vec{B})$

Thus equation (1) can be written as $\frac{dW}{dt} = -\frac{d}{dt}\int_V \frac{1}{2}\left(\varepsilon_0E^2 + \frac{1}{\mu_0}B^2\right)dV - \frac{1}{\mu_0}\int_V \nabla.(\vec{E}\times\vec{B})dV$

Form divergence theorem $\int_V \nabla.(\vec{E}\times\vec{B}) = \oint_S (\vec{E}\times\vec{B})\overrightarrow{da}$

Thus $\frac{dW}{dt} = -\frac{d}{dt}\int_V \frac{1}{2}\left(\varepsilon_0E^2 + \frac{1}{\mu_0}B^2\right)dV - \frac{1}{\mu_0}\oint_S (\vec{E}\times\vec{B})\overrightarrow{da}$ ....(5)

The above equation is the Poynting theorem. The first integral on the right is the total energy stored in the field given by $U_{em} = \frac{1}{2}\left(\varepsilon_0E^2 + \frac{1}{\mu_0}B^2\right)$ and the second term is the rate at which energy is carried out of V called the pointing vector given by $\boldsymbol{\vec{S}} = \frac{\mathbf{1}}{\boldsymbol{\mu_0}}\boldsymbol{\vec{E}}\times\boldsymbol{\vec{B}}$.

It is the energy per unit time per unit area transported by the electromagnetic fields called the **poynting vector**. It is also called energy flux density. The unit of poynting vector is m^{-2} . Electromagnetic waves travel in the direction of poynting vector and transport energy.

The pointing theorem given by (5) states that, the work done on the charges by the electromagnetic force is equal to the decrease in energy stored in the field minus the energy that flowed out through the surface.

Thus $\frac{dW}{dt} = -\frac{dU_{em}}{dt} - \oint_S \overrightarrow{S da}$.

Energy density

The energy per unit volume stored in the electromagnetic field is
$U_{em} = \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2\right)$. As $= \frac{E}{c}$, we have
$U_{em} = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0}\frac{E^2}{c^2} = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0} E^2 \mu_0 \varepsilon_0$ Here $c^2 = \frac{1}{\mu_0 \varepsilon_0}$
Hence $U_{em} = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\varepsilon_0 E^2 = \varepsilon_0 E^2$. Similarly it can be shown that $U_{em} = \frac{B^2}{\mu_0}$
.Hence total instantaneous energy density $U_{em} = \varepsilon_0 E^2 = \frac{B^2}{\mu_0}$.
Hence the average energy density associated with electric field is equal to that of magnetic fields. i.e. energy is shared equally by the two fields.

Note :

1. The poynting vector is $\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$ Also $\vec{E} \times \vec{B} = EB$
Thus magnitude of poynting vector is $= \frac{EB}{\mu_0}$,
As $B = \frac{E}{c}$, we can express poynting vector as $S = \frac{E^2}{\mu_0 c} = \frac{c\,B^2}{\mu_0}$.
The above equations represent the instantaneous rate at which energy is passing through a unit area.
2. The energy incident per unit time on a unit area of surface perpendicular to the propagation of wave us called intensity of the wave. It can be sown that the intensity expression is $I = \frac{1}{2}\varepsilon_0 c E_0^2 + \frac{1}{2\mu_0} c B_0^2$.

Momentum and pressure of electromagnetic waves

Electromagnetic waves transport linear momentum in space in addition to the transport of energy. Therefore it is possible to exert pressure i.e., radiation pressure on a surface when the electromagnetic waves incident on it.
If an electromagnetic wave of energy U is propagating with the speed c, then the total momentum transported to a surface due to complete absorption is $p = \frac{U}{c}$.
By definition, pressure exerted on a surface as the force per unit area. $P = \frac{F}{A}$.
Also from, Newton's second law $= \frac{dp}{dt}$.
Thus the pressure is $P = \frac{F}{A} = \frac{1}{A}\frac{dp}{dt} = \frac{1}{A}\frac{d}{dt}\left(\frac{U}{c}\right)$
Thus $= \frac{F}{A} = \frac{1}{A}\frac{dp}{dt} = \frac{1}{c}\left(\frac{dU/dt}{A}\right)$. In this equation $\left(\frac{dU/dt}{A}\right)$ represents the rate at which the energy is arriving at the surface per unit area. It is called the magnitude of poynting vector. Thus the radiation pressure P on a perfectly absorbing surface is$P = \frac{S}{c}$.
The momentum of a closed system consisting of a field and the particles (mechanical) is conserved which is the law of conservation of electromagnetic

field momentum. Accordingly $\frac{\partial \vec{p}_{mech}}{\partial t} + \frac{\partial \vec{p}_{field}}{\partial t} = \nabla . \overleftrightarrow{T}$ where $\overleftrightarrow{T}$ is the stress tensor which is the force per unit area (stress) acting on the surface.
The momentum density is given by $\vec{p}_{field} = \int_V (\vec{D} \times \vec{B}) dV$ where $\vec{g} = (\vec{D} \times \vec{B})$ is called the electromagnetic momentum density. It is related to poynting vector as $\vec{g} = (\varepsilon \vec{E} \times \vec{B}) = \frac{\mu\varepsilon}{\mu}(\vec{E} \times \vec{B}) = \mu\varepsilon \vec{S}$.

Electromagnetic waves in a conducting medium
Consider a conducting medium free from charges. Thus $\rho = 0$The current density $\vec{J} = \sigma\vec{E}$ .Here σ is the conductivity of the medium. The medium has absolute permittivity ε and absolute permeability as μ . Thus the electric displacement is $\vec{D} = \varepsilon \vec{E}$.
The four Maxwell's equations are
$\nabla . \vec{D} = \rho$, $\nabla . \vec{B} = 0$, $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$\nabla \times \vec{B} = \mu\left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)$
By applying the above conditions to Maxwell's equations, we get
$\nabla . \vec{E} = 0$ .... (1) $\nabla . \vec{B} = 0$ ......(2)
$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ ......(3) $\nabla \times \vec{B} = \mu\left(\sigma\vec{E} + \varepsilon\frac{\partial \vec{E}}{\partial t}\right)$ .....(4)
Taking curl on both sides of equation (3) $\nabla \times \nabla \times \vec{E} = -\nabla \times \frac{\partial \vec{B}}{\partial t}$
or $\nabla \times (\nabla \times \vec{E}) = -\frac{\partial(\nabla \times \vec{B})}{\partial t}$ ......(5)
substituting for $\nabla \times \vec{B}$ from (4) in (5), we get $\nabla \times (\nabla \times \vec{E}) = -\frac{\partial}{\partial t}\left(\mu\sigma\vec{E} + \mu\varepsilon\frac{\partial \vec{E}}{\partial t}\right)$
or $\nabla \times (\nabla \times \vec{E}) = -\mu\sigma\frac{\partial \vec{E}}{\partial t} - \mu\,\varepsilon\frac{\partial^2 \vec{E}}{\partial t^2}$ ......(6)
But $\nabla \times (\nabla \times \vec{E}) = \nabla(\nabla . \vec{E}) - \nabla^2\vec{E}$
Thus equation (6) is $\nabla(\nabla . \vec{E}) - \nabla^2\vec{E} = -\mu\sigma\frac{\partial \vec{E}}{\partial t} - \mu\,\varepsilon\frac{\partial^2 \vec{E}}{\partial t^2}$
From equation (1) $\nabla . \vec{E} = 0$, thus the above equation becomes

$$-\nabla^2\vec{E} = -\mu\sigma\frac{\partial \vec{E}}{\partial t} - \mu\,\varepsilon\frac{\partial^2 \vec{E}}{\partial t^2}$$

or $\boldsymbol{\nabla^2\vec{E} = \mu\sigma\frac{\partial \vec{E}}{\partial t} + \mu\,\varepsilon\frac{\partial^2 \vec{E}}{\partial t^2}}$ ....(7)
Repeating the above process by taking the curl on both sides of equation (4),
$\nabla \times (\nabla \times \vec{B}) = \nabla \times \mu\left(\sigma\vec{B} + \varepsilon\frac{\partial \vec{B}}{\partial t}\right)$ or $\nabla \times (\nabla \times \vec{B}) = \mu\sigma(\nabla \times \vec{B}) + \mu\varepsilon\frac{\partial(\nabla \times \vec{B})}{\partial t}$
substituting for $\nabla \times \vec{E}$ from (3) the above equation,

$$\nabla \times (\nabla \times \vec{B}) = \mu\sigma\left(-\frac{\partial \vec{B}}{\partial t}\right) + \frac{\partial}{\partial t}\left(-\mu\,\varepsilon\frac{\partial \vec{B}}{\partial t}\right)$$

But $\nabla \times (\nabla \times \vec{B}) = \nabla(\nabla . \vec{B}) - \nabla^2\vec{B}$

Thus we have $\nabla(\nabla . \vec{B}) - \nabla^2\vec{B} = -\mu\sigma\left(\frac{\partial\vec{B}}{\partial t}\right) - \mu\,\varepsilon\frac{\partial^2\vec{B}}{\partial t^2}$

From equation (2) $\nabla . \vec{B} = 0$, thus the above equation becomes

$-\nabla^2\vec{B} = -\mu\sigma\left(\frac{\partial\vec{B}}{\partial t}\right) - \mu\,\varepsilon\frac{\partial^2\vec{B}}{\partial t^2}$ or $\boldsymbol{\nabla^2\vec{B} = \mu\sigma\left(\frac{\partial\vec{B}}{\partial t}\right) + \mu\,\varepsilon\frac{\partial^2\vec{B}}{\partial t^2}}$....(8)

Equations (7) and (8) are called the electromagnetic wave equations in a conducting medium. These equations represents a damped wave motion where the amplitudes $\vec{E}$ and $\vec{B}$ decreases as the wave progresses. This happens due to the existence of the extra term $\frac{\partial\vec{E}}{\partial t}$ and $\frac{\partial\vec{B}}{\partial t}$ in the wave equations.

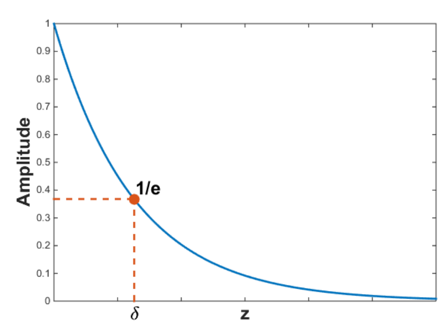
Skin effect : The non uniform distribution of electric current over the surface or the skin of the conductor carrying AC is called skin effect. In other words the concentration of charge is more near the surface as compared to the core of the conductor. The ohmic resistance of the conductor is increased due to the concentration of current on the surface of the conductor.

The skin effect increases with increase in frequency. At low frequency, such as 50 Hz, there is a small increase in the current density near the surface of the conductor but at high frequencies the whole current flows on the surface of the conductor. If DC is passed through the conductor, as frequency is zero, current is uniformly distributed over the cross section of the conductor. This is due to larger magnetic flux linking the cylindrical elements near the centre of the conductor as compared to that at the surface. This results in greater inductive reactance at the centre than at the surface resulting in more current at the surface.

Skin depth

Skindepth is defined as the depth at which the amplitude of the wave has been reduced by 1/e or 37% of its original value.

Attenuation : Attenuation defines the rate of amplitude loss an EM wave experiences at it propagates (graph).



The attenuation of an EM wave is defined by the parameter α called attenuation constant. The reciprocal of attenuation constant is called the skin depth given by $\delta = \frac{1}{\alpha}$.

Consider the wave attenuation as represented by the equation $E = E_0 e^{-\alpha x}$ where x is the distance of propagation of the wave in the medium.

If $x = \delta$, then $E = E_0 e^{-\alpha\,\delta}$

Also, by definition $\delta = \frac{1}{\alpha}$, then $E = E_0 e^{-1} = \frac{E_0}{e} = 0.37E_0$.

For good conductors, the depth of penetration is given by $\delta = \frac{1}{\alpha} = \sqrt{\frac{1}{\omega\mu\sigma}}$.

For example, the depth of penetration in case of copper for a wave frequency f =1MHz, (also $\omega = 2\pi f$) with the thermal conductivity of copper as $\sigma = 5.6 \times 10^7$ Sm^{-1} and the permeability in free space as $\mu_0 = 4\pi \times 10^{-7} Hm^{-1}$, we get $\delta = 0.0667\, mm$.

PART A

1. Explain gradient of a scalar function, Divergence and Curl of a vector function. Give geometric representation of divergence and curl.
2. (a) Explain divergence theorem and stoke's theorem.
   (b) Arrive at relation between electric field and potential.
3 (a) Explain electric displacement.
   (b) What is equation of continuity? Derive the equation of continuity.
4. Derive the following Maxwell's equations $\nabla . \vec{D} = \rho$ and $\nabla \times \vec{E} = -\frac{\partial B}{\partial t}$
5. Derive the following Maxwell's equations $\nabla . \vec{B} = 0$ *and* $\nabla \times \vec{B} = \mu_0 \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)$
6. Explain the significance of Maxwell's equations.
7. (a) Arrive at the electromagnetic wave equation in free space.
   (b) Comment on speed of electromagnetic waves.
8 (a) Show that the electromagnetic waves are transverse in nature.
   (b) Arrive at the electromagnetic wave equation for a wave propagating in a dielectric medium and hence relate the refractive index with permittivity of the medium.
9. (a) What is pointing vector? Give the significance of Poynting vector.
   (b) Show that electromagnetic waves are transverse in nature. Arrive at the relation between electric and magnetic vector.
10. Arrive at the expression for energy carried by a electromagnetic wave. Hence state Poynting theorem.
11. Write a note on momentum carried by electromagnetic waves and arrive at the expression for the momentum. What is radiation pressure? Explain.
12. (a) Arrive at the expression for wave equation for a wave propagating in a conducting medium.
   (b)Write a note on skin effect and skin depth.

PART B

1 Find the divergence of a vector $A = x^2 z\,\hat{\imath} + 2y^2 z^2 \hat{\jmath} + xy^2 z\hat{k}$ at point (1,-1,1).
Hint : $\nabla . \mathrm{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} = \frac{\partial x^2 z}{\partial x} + \frac{\partial 2y^2 z^2}{\partial y} + \frac{\partial xy^2 z}{\partial z} = 2xz + 4yz^2 + xy^2 = 2 - 4 + 1 = -1$ (put x =1, y = -1, z = 1)

2 Describe the electric field that go with the following potential V =x^2+$y^2 + z^2$.
Hint : $E = -\nabla \mathrm{V} = -\frac{\partial x^2}{\partial x}\hat{\imath} - \frac{\partial y^2}{\partial y}\hat{\jmath} - \frac{\partial z^2}{\partial z}\hat{k} = -2x\,\hat{\imath} - 2y\,\hat{\jmath} - 2z\,\hat{k}$

3 Calculate the curl and the divergence of the following vector field. $F_x = x + y,\ F_y = -x +$

$y, F_z = -2z$. Hint : $\nabla \times \vec{F} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\hat{\imath} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\hat{\jmath} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\hat{k} = 0 + 0 + (-1-1)\hat{k} = -2\hat{k}$$\nabla . \mathrm{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} = 1 + 1 - 2 = 0$

4 Find the value of constant c for which the vector $A = (x+3y)\hat{\imath} + (y-2z)\hat{\jmath} + (x+cz)\,\hat{k}$ is solenoidal.

5 The voltage between the plates of a parallel plate capacitor of capacitance 1μF is changing at the rate of 5Vs^{-1}. What is the displacement current in the capacitor?

Hint: In the equation $\nabla \times \vec{B} = \mu_0\left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)$, $I_D = \frac{\partial \vec{D}}{\partial t}$is called displacement current$I_D = C\frac{dV}{dt} = 1 \times 10^{-6} \times 5 = 5\mu A$

6 A plane electromagnetic wave travelling along X-direction in an unbounded lossless dielectric medium of $\mu_r = 2$ and $\varepsilon_r = 5$has peak electric field strength of 10 Vm^{-1}. Calculate (i) the velocity of the wave (ii) refractive index and (iii) average energy density. Hint: $v = \frac{c}{\sqrt{\mu_r \varepsilon_r}}$, $n = \sqrt{\mu_r \varepsilon_r}$$U = \varepsilon_0 E_{av}^2$ where $E_{av} = \frac{E_0}{\sqrt{2}}$.

7 A plane electromagnetic wave in the visible region is moving along +X direction. The frequency of the wave is $0.5 \times 10^{15} Hz$ and the electric field at any point is varying sinusoidally with time with an amplitude of 1 Vm^{-1} . Calculate the instantaneous values of the densities of the electric and magnetic fields

$\varepsilon_0 = 8.854 \times 10^{-12} Fm^{-1}$ and $\mu_0 = 4\pi \times 10^{-7} Hm^{-1}$.

Hint : $U_E = \frac{1}{2}\varepsilon_0 E^2$ where $E = \frac{E_0}{\sqrt{2}}$, $E_0 = 1Vm^{-1}$, $B = \frac{E}{c}$ and $U_B = \frac{1}{2}\frac{B^2}{\mu_0}$

8. An electromagnetic wave in free space has a wavelength of 0.2 m. When he same wave enters a dielectric medium, the wavelength changes to 0.09 m. Assuming that $\mu_r = 1$, determine ε_r and the velocity of the wave in the dielectric medium.

Hint: $f = \frac{c}{\lambda_1}$ where $\lambda_1 = 0.2\,m$, In dielectric medium $v = f\lambda_2$ where $\lambda_2 = 0.09m$ $v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_0 \varepsilon_0 \mu_r \varepsilon_r}}$ or $\varepsilon_r = \frac{1}{\mu_0 \varepsilon_0 \mu_r v^2}$

9 Calculate the skin depth in copper of conductivity 5.6×10^7 Sm^{-1} for the electromagnetic waves of frequency 1MHz and the permeability in free space as $\mu_0 = 4\pi \times 10^{-7} Hm^{-1}$,

10 An ac voltage is applied directly across a 10 μF capacitor. The frequency of the source is 3 KHz and the voltage amplitude is 30V. Find the displacement current between the plates of the capacitor.

PART C

1 If the divergence of a vector at a point is positive, the point is called source. Explain

Ans: As the flux of the vector flows out of the source, the point is called a source.

2 Light is electromagnetic in nature. Explain why?

Ans: Light waves are oscillating electric and magnetic field vectors perpendicular to each other. Thus light is electromagnetic in nature.

3 Light rays, X-rays, γ -rays are not deflected by electric and magnetic fields. Yet they are called electromagnetic waves. Explain why?

Ans : They do not carry any electric charges and the fields produced are induced fields. Thus they are not deflected by electric or magnetic fields.

4 velocity of electromagnetic wave in a dielectric medium is less than that in free space. Explain why?

Ans :When an electromagnetic wave enters a material medium, the atoms absorb and reemit the waves with the same frequency but this process causes the net speed to decrease.

5. Ampere's circuital law is valid for steady state phenomenon and not for changing fields. Why?
   Ans: This law inconsistent with the equation of continuity It does not agree with law of conservation of charges when there is decrease or increase of charges. Thus it is not applicable to changing fields.
6. Is electromagnetic wave transverse? Explain.
   Ans: Yes. Electromagnetic waves are transverse in nature as the electric and magnetic field vectors oscillate perpendicular to each other and also perpendicular to the direction of wave propagation.
7. What is the basic source of electromagnetic waves? Explain.
   Ans: An accelerating electric charge is the source of electromagnetic waves. The induce electric fields and which in turn induce magnetic fields and vice versa which propagate in vacuum or material media.
8. If the divergence of vector field is zero, the field is called solenoidal. Explain.
   Ans: If divergence is zero means no net flux can occur across any closed surface, i.e. the flux lines are closed loops without any source or sink. Thus thet are referred to as curl or solenoidal.
9. What is the physical significance of the equation $\nabla . B = 0$?
   Ans: This indicates that divergence of magnetic field is zero, i.e. flux lines neither move outwards from a point or converge to a point. This shows there are no magnetic monopoles exists. They exists as dipoles only.
10. Is it possible to have only electric wave or magnetic wave propagating through space? Explain.
    Ans: No. An electric wave induces a magnetic wave and a magnetic wave induces electric wave. i.e. a changing electric field produces changing magnetic field and vice versa.